



Are viscosity ratios of rocks measurable from cleavage refraction?

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Abstract

In this *Questions* contribution, I consider some of the accepted methods of measuring viscosity ratios in rocks, and pose a new one. For a wide range of strain and orientations, it can be shown that the bedding normal is immeasurably close to the XY plane, and this has applications to the relationship between cleavage and strain. I therefore propose that with some limitations, cleavage refraction can provide a measure of effective viscosity ratios in layered rocks. Examples show that cleavage refraction across competence contrasts yields surprisingly small viscosity ratios. This method might also provide a way of distinguishing Newtonian from non-Newtonian behaviour of rocks over time and space. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

When geological structures such as folds are modelled, theoretically, numerically or with analogue materials, the competence contrast is quantified as the *viscosity ratio* of layer to matrix. For linear-viscous materials (Newtonian), this ratio is a constant: but for non-linear materials, viscosity ratios will be dependent on strain rate, and perhaps other system variables. In multilayers with different power-law properties, the viscosity contrasts are not simply dependent on the component layers, but also on the type of bulk flow and/or the orientations of the layers (Treagus, 1993); they might therefore be expected to vary throughout a deformation.

The viscosity of a fluid or rock strictly only describes the properties governing instantaneous flow; and likewise, the viscosity ratios. However, in geological terms and time scales, we need to conceive of a finite *effective viscosity* that describes the properties of total flow, integrated over a certain interval of natural deformation. The viscosity ratios assumed or determined for geological structures, or their modelling, would be defined this way, and are qualitatively

termed competence or ductility contrasts (Ramsay, 1982).

Many types of geological structures provide indications of competence or viscosity contrasts: for example, folds, boudins, mullions, and cleavage refraction (Ramsay, 1982). Such structures allowed Ramsay to construct an 'order of competence' for suites of rocks deformed under similar metamorphic conditions. How might such a scale of competence be converted into numerical viscosity ratios?

1.1. Ways of quantifying viscosity ratios in rocks

1.1.1. Folds

Viscosity ratios in rocks have been deduced from wavelength–thickness ratios of folds (e.g. Sherwin and Chapple, 1968; Hara and Shimamoto, 1984), by application of single-layer buckling theory to rocks. The complexities of various buckling theories cannot be fully reviewed here, except to note that when refinements such as layer-parallel shortening and/or non-Newtonian rheology are included, there are then so many variables that viscosity ratios cannot be determined for a particular wavelength–thickness ratio without prior assumptions. Using two variants of buckling theory, Hara and Shimamoto (1984) calculated viscosity ratios (from single-layer folds) of 94–147 for quartz vein–pelitic schist, 23–40 for quartz

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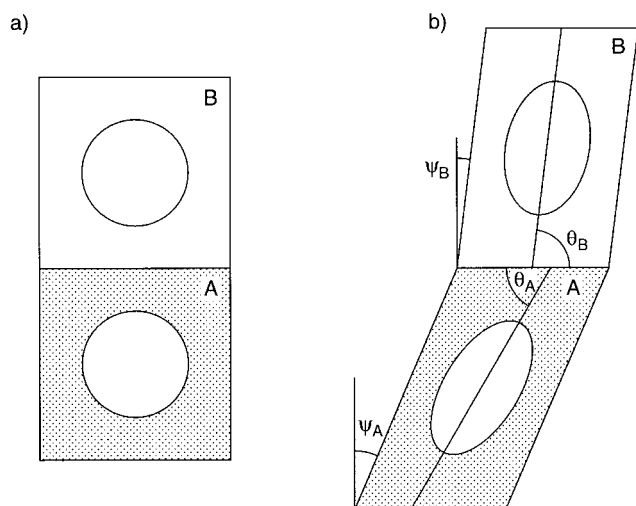


Fig. 1. Strain refraction from layer A to layer B with viscosity ratio 5, after Treagus (1983, 1988). (a) Undeformed square sections of two layers. (b) Deformed states, with parallelograms showing components both of layer-parallel strain and shearing (ψ). The strain refraction is given by $\tan \psi_A / \tan \psi_B = 5$. The orientations of long axes of the strain ellipses to layering are θ_A and θ_B .

vein–psammitic schist, and 14–32 for quartz vein–basic schist. (A psammitic to pelitic schist might, by deduction, be estimated to be in the range from 2.4 to 6.) The greatest problem in application of this approach, is that most natural folds are not single-layered, but multilayered.

1.1.2. Conglomerates

Another way of trying to quantify viscosity ratios comes from the study of deformed conglomerates. This may be idealised into the theory of deformed ellipsoidal objects of different viscosity from a matrix (e.g. Gay, 1968a,b; Bilby et al., 1975). Lisle et al. (1983) applied such theory to deformed conglomerates. While reporting a wide spread of data which disallow precision in either strain measurements or computation of viscosity ratios, they derived average pebble to matrix viscosity ratios of 9 for quartz, 2–1.4 for leptonite, and 0.9 for shale. I am currently developing comparable research on deformed non-ellipsoidal objects (Treagus et al., 1996), and their natural occurrence in deformed fragmental rocks (work in progress with J.E. Treagus), to be reported elsewhere.

1.1.3. Strain variations

A third method of determining viscosity ratios is from strain variations in rocks, as modelled theoretically (Treagus, 1983, 1988, and references therein) and in analogue experiments (Treagus and Sokoutis, 1992). Fig. 1 illustrates theoretical strain refraction and variation across a bonded boundary with viscosity ratio of 5. Fundamental to this analysis is the rule that the shear strain-rate ratio across a boundary is equal to

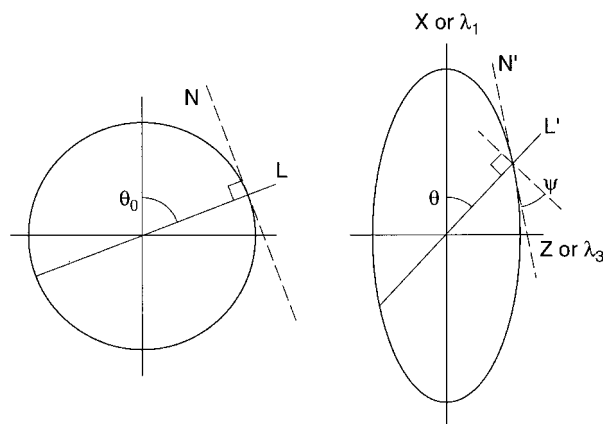


Fig. 2. Two-dimensional strain of a circle to an equal-area ellipse, showing changes in orientation of an arbitrary line (L) and its normal (N), the angular shear (ψ), and orientation (θ) to the maximum extension (X or λ_1).

the inverse viscosity ratio. This leads to a more general rule for finite strain (Treagus, 1983): *the shear strain ratio across a boundary is the inverse viscosity ratio:*

$$\gamma_A / \gamma_B = \tan \psi_A / \tan \psi_B = \mu_B / \mu_A. \quad (1)$$

The deformation variations (Fig. 1) can be considered in terms of an equal component of layer-parallel/perpendicular strain, and a refracting component of shear strain. It follows that where layers are in perfect layer-parallel strain (so $\gamma = 0$), deformation is homogeneous and there is no strain refraction.

To apply Eq. (1) to rocks, we require a feature that was originally layer-normal, to record the shear strains across layer boundaries. Sedimentary dykes or worm burrows, if it can be assumed that these always initiate perpendicular to bedding, would be ideal indicators, particularly if they were seen to refract to provide angles such as ψ_A and ψ_B in Fig. 1. However, sedimentary features such as these are more likely to be confined to one unit, rather than traversing from one rock layer to another, so may rarely provide sufficient data to apply this simple equation.

The most pervasive feature seen through successive rock layers is usually a fanning or refracting cleavage, that might (arguably) be assumed to represent the XY planes of refracting strain ellipsoid in the rocks. From many of the examples of theoretical two-dimensional strain variations given in Treagus (1983, 1988), the X directions are found to be very close in orientation to the deformed layer normals. This leads me to the question: *Are viscosity ratios of rocks measurable from cleavage refraction?*

2. Strain orientation vs shear strain

To attempt an answer to this question, I will first

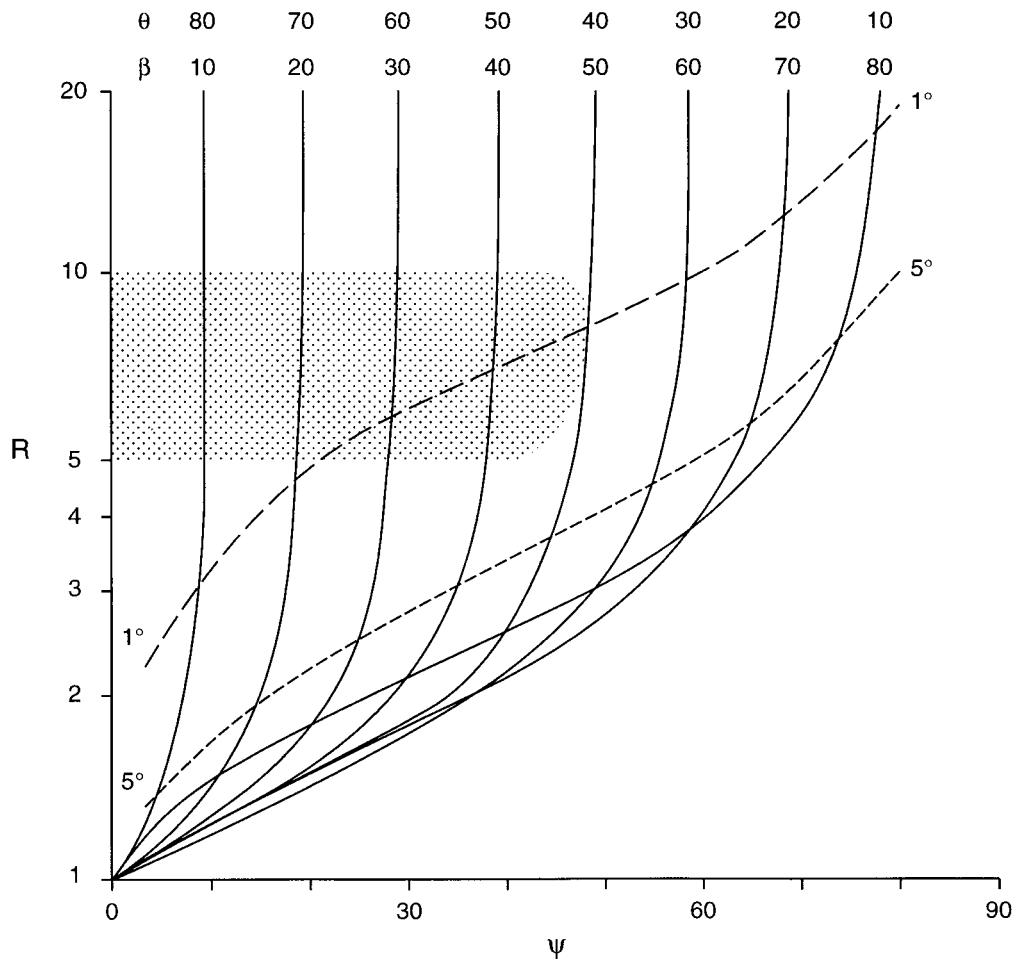


Fig. 3. The relationships of ψ , R , and θ or β , derived from Eq. (4). R is the strain ratio, ψ and θ are defined in Fig. 2, and $\beta = (90^\circ - \theta)$. Solid lines are drawn at 10° intervals of θ and β . Note that ψ is always $< \beta$, but the values become close: see broken curves marking 1° and 5° difference. The graph region above/left of the 1° curve is taken as $\beta \cong \psi$. The R and θ values from published data for slates are shown stippled.

consider the geometry of strain in terms of lines and orientations, without thought of strain refraction or viscosity ratios. The analysis will be two-dimensional and will consider the mutual relationship between angular shear (ψ) of a line (L), and its orientation (θ) in a strain ellipse (Fig. 2). Following the expressions in Ramsay (1967, equations 3.31 and 3.42) for determining reciprocal quadratic elongation (λ') and shear strain ($\gamma = \tan \psi$), for an arbitrary line at θ to the maximum extension (X or λ_1):

$$\lambda' = \lambda'_1 \cos^2 \theta + \lambda'_2 \sin^2 \theta \quad (2)$$

$$\gamma' = \gamma \lambda' = (\lambda'_2 - \lambda'_1) \sin \theta \cos \theta. \quad (3)$$

Consider this arbitrary line (L) to be bedding or layering orientation in strain, so that its angular shear (ψ) is the change with respect to layer-normal, N (Fig. 2). These equations can then be manipulated to derive an expression for ψ in terms of θ and the strain. This

process is simplified by considering equal-area plane strain, so that all the expressions can be given in terms of stretch ratio, R (with $R = 1/\lambda'_1 = \lambda'_2$). It is found that:

$$\tan \psi = (R^2 - 1) \tan \theta / (1 + R^2 \tan^2 \theta). \quad (4)$$

To determine how close the deformed normal, N' , is to the X or λ'_1 axis (Fig. 2), we need to compare ψ with $(90^\circ - \theta)$, which I term angle β . [β angles of X axes or cleavage traces to layer normals are a useful method of analysing strain and cleavage in folds (Treagus, 1982, 1997).]

Fig. 3 shows the relationships of ψ , R , and θ or β , graphically, derived from Eq. (4). β angles are always more than ψ , but for a significant field of strain and ψ or β values, they have very close values (note lines of 1 and 5° difference). The strain values for which $\beta \cong \psi$ (taken as $< 1^\circ$ difference) are $R = 4.6$ for $\beta = 20^\circ$ ($\theta = 70^\circ$), and $R = 7$ for $\beta = 40^\circ$ ($\theta = 50^\circ$). It should be noted that there is no deformation history in the

data in Fig. 3; the relationships are solely those of a strain ellipse. Curves could be added to describe the relationships for particular deformation histories, such as layer-parallel simple shear, or pure shear in a particular orientation.

3. Application to cleavage refraction (and questions about cleavage)

How might the information in Fig. 3 be relevant to strain and cleavage refraction in rocks? Or indeed, to the thornier question of whether it can be assumed that cleavage exactly represents the XY plane of strain (Williams, 1976)? To answer this, we need to consider values of strain and cleavage–bedding angles in layered and folded rocks.

3.1. Slates and slaty cleavage

A strain ratio of $R = 2$ ($\approx 30\%$ shortening) is sometimes taken as a threshold for cleavage formation in mudrocks. Classic strain data from reduction spots in many Cambrian slates (Wood, 1974; fig. 4) yield R ($= X/Z$) values in a wide range from $R = 3$ to 22, with a concentration at 5–10. These rocks have folds yielding cleavage–bedding angles (θ) of 90 – 45° ($\beta = 0$ – 45°). This range of data is stippled in Fig. 3, as typical for slate belts, for comparison with the calculated relationships for ψ and θ or β discussed above. The $R = 5$ – 10 concentration occupies the range where $\beta \cong \psi$, for $\beta = 0$ – 25° (cleavage–bedding angles 65 – 90°). Only for β in excess of 40° (cleavage–bedding angles $< 50^\circ$) do the differences from ψ begin to exceed 2° . However, it is unlikely that angles could be recorded to a 2° accuracy, in the field, and differences even of 5° might not be measurable. [There are analogies, here, to Ghosh's consideration of shear along foliations (Ghosh, 1982).] If this is the case, there is an even greater range of strains and orientations where cleavage and deformed layer-normals might be indistinguishable (Fig. 3; 5° curve).

Cleavage–bedding angles recorded by Gray (1981) in folded rocks from the Appalachians show minima of about 40° for mudrock (and 50° and 60° for limestone and sandstone, respectively). This, again, tends to confirm a maximum β of 50° , placing the data in the field where the deformed bedding-normal and XY plane are virtually indistinguishable.

From these data, it might be *expected* that in slates, original bedding-perpendicular features such as sandstone dykes and worm burrows would *appear* to lie in the cleavage planes. This appearance of parallelism may explain one of the paradoxes concerning cleavage formation: whether cleavage represents material or immaterial planes (Williams, 1976). [Other arguments,

such as cleavage and the 'dewatering' hypothesis will not be expanded here (Siddans, 1972; Wood, 1974; Groshong, 1976, and references therein).]

I conclude that cleavage can be used as an approximation for the deformed bedding normal in slates, at strain ratios of > 5 , and/or bedding–cleavage angles of $> 45^\circ$ ($\beta < 45^\circ$). However, to use cleavage as a bedding-parallel shear strain indicator, to determine a viscosity ratio from Eq. (1), (a) evidence of sharp cleavage refraction across a competence contrast is needed, and (b) the refracted cleavage must *also* be assumed to be an approximation to the deformed bedding-normal. It is therefore necessary to ask whether cleavage of other morphologies (e.g. spaced) either originated as a bedding-perpendicular feature, or can be approximated as such.

3.2. Other rocks and cleavages

In my earlier reviews of the relationship of cleavage to strain for other cleavage morphologies (Treagus, 1983, 1988), such as spaced cleavage, I concluded there was little strain data available to prove or disprove parallelism to XY planes. Strain is difficult to measure in weakly deformed competent rocks, and where cleavages are material seams such as pressure-solution surfaces, strain may be quite localized within the rock. Nevertheless, it is difficult to avoid the conclusion that most spaced cleavages must be material surfaces.

Geiser (1974) and Groshong (1976) concluded that their pressure-solution spaced cleavages originated perpendicular to bedding. Likewise, Yang and Gray (1994) conclude that their cleavage in folded sandstones originated perpendicular to bedding, in early layer-parallel shortening. This raises two questions as alternatives. (1) Is it a feature of competent rocks, for principal strain axes to be oriented nearly layer-parallel/perpendicular? This is demonstrated by my results on strain refraction (cited earlier). (2) Is it a feature of spaced cleavages, to form subperpendicular to bedding, even where principal strains are not? This appears to be the case in rocks which have two cleavages, one representing material surfaces, the other a grain-shape fabric (Boulter, 1979; Yang and Gray, 1994). Although I cannot address all these questions about cleavage in this article, it might be reasonable to conclude that spaced cleavages in competent rocks are a better indicator of initial layer-normal material planes (perhaps subsequently rotated and deformed in folding), than they are of XY planes of total or tectonic strain. If this is indeed true, they provide the required bedding-shear indicator from which to calculate a viscosity ratio.

3.3. Verdict

For several different reasons, it is concluded that

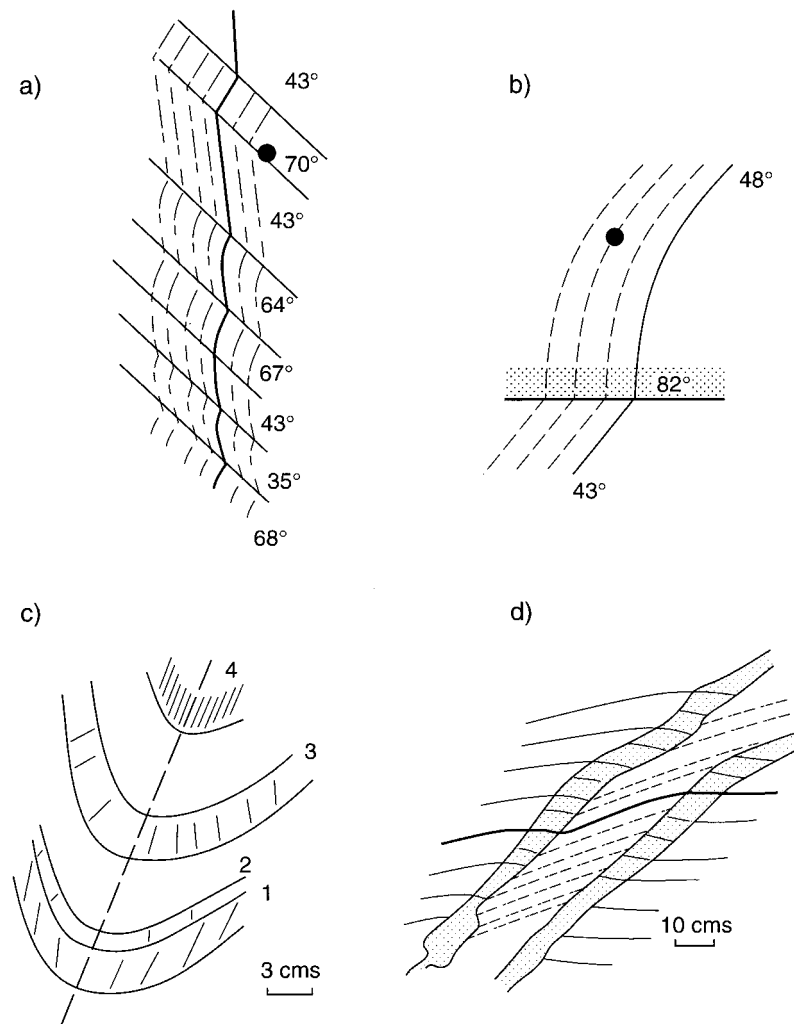


Fig. 4. Four examples of cleavage refraction that might be used to estimate viscosity ratios in rocks. (a) Cleavage refraction (dashed lines) across beds of greywackes and slates, drawn from Ramsay (1967, fig. 7-71), including Ramsay's own trajectory (bold line). Bedding–cleavage angles (θ) are shown. Black circle is coin scale. (b) Cleavage refraction in graded psammite (stippled base) to pelite, and cleavage–bedding angles, drawn from Ramsay and Huber (1983, fig. 10.22). Black circle is coin scale. (c) Cleavage fanning and refraction in folded psammites and pelites (measured beds numbered from 1 to 4), after Treagus (1982, fig. 6). (d) Schistosity refraction from a gneiss into a composite dyke with a quartzo-feldspathic fringe (stippled) and biotized amphibolite inner region (shown by dashed foliation traces), drawn after Sengupta (1997, fig. 17.6), with a typical trajectory shown in bold.

refracting first cleavages across lithological sequences may provide an approximate measure of shear strain variations, and therefore may be used to measure viscosity ratios. So instead of the simple Eq. (1), given earlier as:

$$\gamma_A/\gamma_B = \tan \psi_A/\tan \psi_B = \mu_B/\mu_A,$$

an equation can now be written in terms of β :

$$\tan \beta_A/\tan \beta_B = \mu_B/\mu_A, \quad (5)$$

or in terms of cleavage–bedding angles (θ):

$$\tan \theta_A/\tan \theta_B = \mu_A/\mu_B. \quad (6)$$

This last equation is a tentative ‘answer’ to the ques-

tion posed at the beginning. Its restrictions and potential applications are considered next.

4. Discussion and examples

The simple relationship of orientations of strain axes or fabrics to viscosity ratio given in Eq. (6), and with certain restrictions, has a potentially wide application in structural geology. The restrictions have been discussed above in terms of slates, and also by testing Eq. (6) for the theoretically derived strain refraction patterns in Treagus (1983, 1988). These are summarized as follows.

1. The layering should be in the shortening field of finite strain.
2. Cleavage–bedding angles should ideally be more than 45°: but the method may work for values down to about 30° if strain is high enough.
3. The method seems most amenable to viscosity ratios between 1 and 10 (in contrast to calculations from buckling analysis).
4. The analysis would break down for extremely competent rocks, where strain would be negligible, and cleavage might not be formed. For extremely incompetent rocks, cleavage–bedding angles might be too small: see point 2.

With these restrictions in mind, I consider some examples in Fig. 4, taken from published sources. Fig. 4(a) is an example of *slaty cleavage refraction*, from Ramsay (1967, fig. 7-71). Cleavage–bedding angles are shown, yielding pseudo-viscosity ratios among greywacke and slate (with respect to the 43°-cleaved layer) of 2.9, 2.2, 2.5, 0.75 and 2.8, from top to bottom. An example of *curved cleavage refraction* in graded units is given in Fig. 4(b), from Ramsay and Huber (1983, fig. 10.22). The viscosity ratio of the psammite base to pelitic units is found to be 6.4. The example in Fig. 4(c) (after Treagus, 1982, fig. 6) shows the potential of the method for *folded rocks*. (This fold was formerly used to illustrate a cleavage classification of folds with a β plot.) The cleavage–bedding angles around the fold yield viscosity ratios of layers, with respect to layer 4, of 0.6–0.9 for bed 1, 1.6–1.7 for bed 2, and 2.2–2.7 for bed 3. The maximum psammite to pelite ratio is therefore 2.5–4.0.

If this is a viable method for measuring viscosity ratios in folded rocks, it opens up a possible new method for distinguishing linear from non-linear rheology. It was emphasised at the beginning that only if rocks behave as Newtonian fluids can we expect a constant viscosity ratio between adjacent rock layers, throughout a structure. For example, if the layers in Fig. 4(c) behaved as power-law fluids (recalling that effective viscosity ratios would vary according to strain rate, deformation history and layer orientation), there is no reason to expect the same viscosity ratios all around the fold, among layers 1–4. I shall be pursuing this line of enquiry in a future research programme on rheology of rocks and structures.

The final example, Fig. 4(d) (after Sengupta, 1997, fig. 17.6), illustrates refraction of schistosity from a gneiss through a dyke and its more competent fringe. I tentatively suggest that the schistosity variations here provide a way of measuring viscosity ratios. The data yield values of 1.5 for the dyke fringe to gneiss host, and 2.4 for dyke fringe to inner dyke. Although the ‘cleavage’ here is a schistosity/gneissosity, and must be considered a different kind of structure from the ‘first’

cleavages in the three previous examples, the high strain associated with schists and gneisses may make the method valid. Whether the method might also be applicable to crenulation cleavage variations across competence contrasts will be left open, for future work.

From the first three examples, concerning sedimentary rocks with first cleavage, the pseudo-viscosity ratios might seem surprisingly small: in the order of 6 as a maximum psammite/pelite ratio, and from 2.5 to 4 for another. However, these are a similar magnitude to values quoted in Section 1, from single-layer folds or conglomerates. Many of the ‘competence contrasts’ in Fig. 4 are translated into viscosity ratios of only about 2. It would be premature to speculate widely, at this point, on what this means for the rheology of multilayered sedimentary rocks in general—before this is accepted as a ‘tried and tested’ method. Yet, in the spirit of this celebration of 20 years of the Journal, I would like to end with two points of speculation. (a) It appears that trivially small viscosity ratios can generate measurable strain variations, cleavage refraction and ‘competence contrast’. So perhaps rocks are more similar in their effective viscosities than might be thought. (b) Would the effective viscosity ratios be so small, if sedimentary rocks behaved as power-law fluids: for example with stress exponents of 3 or 5? I think not, but a full answer needs more research.

5. Conclusions

1. In layered rocks with commonly found ranges of strain and cleavage–bedding orientation, cleavage may be immeasurably close to the deformed bedding-normal.
2. Cleavage refraction patterns in rocks may provide a simple method of measuring effective viscosity ratios of rocks, with certain restrictions.
3. According to this method, cleavage refraction across competence contrasts yields surprisingly small viscosity ratios (e.g. 2 or 4).
4. This approach may provide a new way of distinguishing Newtonian from non-Newtonian behaviour of rocks, over time and space.

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